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NOTE ON THE CONTINUED PRODUCT OF THE OPERATORS OF ANY GROUP OF FINITE ORDER

BY WILLIAM B. FITE

MILLER has shown* that if an abelian group contains more than one operator of order two, the continued product of all its operators is the identity; and that if it contains only one such operator, this operator is the continued product of all the operators. In this note I show by the most elementary considerations that non-abelian groups of finite order possess an analogous property. So far as I am aware no one has ever called attention to this fact.

Let G be a non-abelian group of odd order. We can arrange its operators in such a way that every operator is followed by its inverse, and the continued product in this order is the identity. Hence the continued product of these operators in any order is an operator of the commutator subgroup. Moreover, the operators can be arranged so that the product is any given commutator t. For, if $A_i^{-1} A_j A_i = A_j t$, then

$$A_i^{-1} A_i A_j A_j^{-1} \cdots A_n = 1,$$

and
$$A_j^{-1} \cdots A_n A_i^{-1} A_i A_j = 1.$$

Consequently
$$A_j^{-1} \cdots A_n A_i^{-1} A_j A_i = t.$$

Suppose now that G is of even order and that K, of order k, is its commutator subgroup. Since the quotient group G/K is abelian, the continued product of its operators is the identity or an operator of order two. If now we arrange the operators of G by taking first one operator from each division of G/K in turn, then another operator from each of these divisions in the same order, and so on until we have all the operators of G, the continued product of these operators in this order corresponds to the k^{th} power of the product of the operators of G/K. This latter is the identity unless k is odd and G/K has only one operator of order two. In the exceptional case, the product of the operators of G taken in a certain order corresponds to the operator of order two in the quotient group G/K. If we take the operators in any other order, the only effect on the product is to multiply it by some operator of the commutator subgroup. Hence :—

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* ANNALS OF MATHEMATICS, ser. 2, vol. 4 (1903), p. 189. See also *Giornali di Matematiche di Battaglini*, vol. 41 (1903), pp. 203, 336.

The continued product in any order of the operators of a group G whose commutator subgroup is K, of order k, is an operator of K, except when k is odd and the quotient group G/K has only one operator of order two ; in this exceptional case the continued product of the operators of G corresponds to the operator of order two of G/K.

In particular, if G is of class l and order $2^a m$, where m is odd, the product of its operators in any order is an operator of the commutator subgroup, except when the subgroup of order 2^a is cyclic.

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